Physics 123
Experiment 3: Two Dimensional Motion

January 26, 2003

By this time in the lecture portion of the course, you will have done many theoretical problems in which you analyze two-dimensional motion. In this laboratory, you will make measurements on a projectile in free-fall and an object in uniform circular motion.

1 Projectile Motion

When an object is in free fall, it is possible to calculate its trajectory using the kinematic equations of motion in two-dimensions. These equations are a result of the relationship between the acceleration, velocity and positional variables:

\[ \mathbf{\dot{r}} = \frac{d}{dt} \mathbf{r} \]  
\[ \mathbf{\ddot{r}} = \frac{d}{dt} \mathbf{\dot{r}} \]  

In two dimensions, with the y-axis in the vertical direction, this leads to the four kinematic equations:

\[ x = x_0 + v_{ox} t \]  
\[ v_x = v_{ox} \]  
\[ y = y_0 + v_{oy} t - \frac{1}{2} gt^2 \]  
\[ v_y = v_{oy} - gt \]  

In the typical projectile motion problem the initial velocity, \( \mathbf{v}_0 \), is at an angle \( \theta \) to the horizontal axis.

2 Uniform Circular Motion

Another example of motion in two dimensions is that of circular motion. The simplest case is when the radius and velocity of the moving body are held constant. However, even though the speed is constant, the direction of the velocity is continually changing and the body is, therefore, always undergoing acceleration. Imagine a body, originally at some point on a circle designated by radius \( r_0 \) and angle \( \theta \), with a velocity, \( \mathbf{v} \), tangential to the circle (see Figure 1). Its position may be written as:

\[ \mathbf{r} = r_0 \cos \theta \mathbf{i} + r_0 \sin \theta \mathbf{j} \]  

Since the particle is under uniform circular motion, the angle \( \theta \) changes with time as:

\[ \theta = \omega t \]  

where \( \omega \) is the (constant) angular velocity. Thus, combining Equations 7 and 8, we obtain the time dependence of the body’s position.
The velocity and the acceleration of the body are then obtained by applying Equations 9 and 2.

\[
\vec{v} = \vec{r}_o \cos(\omega t) \hat{i} + \vec{r}_o \sin(\omega t) \hat{j}
\]

The velocity and the acceleration of the body are then obtained by applying Equations 9 and 2.

\[
\vec{v} = -\vec{r}_o \omega \sin(\omega t) \hat{i} + \vec{r}_o \omega \cos(\omega t) \hat{j}
\]

\[
\vec{a} = -\omega^2 \vec{r}_o \cos(\omega t) \hat{i} - \omega^2 \vec{r}_o \sin(\omega t) \hat{j}
\]

By looking at Equations 9-11, it is clear that the position, velocity and acceleration vectors all have constant magnitudes as a function of time (\(r_o\), \(\omega r_o\), \(\omega^2 r_o\), respectively). The direction of the acceleration is always perpendicular to the direction of the velocity, which is always perpendicular to the direction of the position vector. The acceleration always points towards the center of the circle.

Since the moving body is experiencing an acceleration towards the center of the circle, then there must be a net force in that direction to keep it under acceleration. By applying Newton’s second law (Note: The equations deal only with the magnitudes, so no vector notation is needed):

\[
F = ma = m \omega^2 r_o = m \frac{v^2}{r_o}
\]

### 3 Experimental Objectives

**Projectile Motion**

In the laboratory you have a Mini Launcher, steel balls, rulers, carbon paper and white paper. The Mini Launcher can be cocked to 3 different positions and therefore release the ball with three different initial velocities. It can also be positioned to any desired angle with the attached protractor. Using this equipment:

- Devise an experimental procedure to determine all three possible initial velocities of the ball released from three different positions of the launcher.
- Devise an experimental procedure to measure the relationship between the range (horizontal distance of the projectile motion) and the angle of release. Use the theoretical relations developed in 1 to develop an expression for the range that you expect for your experimental conditions.

Determine the value for acceleration due to the gravity from this experiment and compare to known tabulated value of 9.81 m/s\(^2\). In order to minimize random errors, it is very important that all of your measurements to be performed several times. Show error bars on your graphs. Make sure that you discuss all possible sources for systematic error in your experiment.

**WARNING:** THE PROJECTILE LAUNCHER CAN SHOOT STEEL BALLS AT HIGH VELOCITIES. IF A PROJECTILE WERE TO HIT YOU IN THE FACE IT COULD CAUSE PERMANENT DAMAGE! ALWAYS WEAR SAFETY GOGGLES
WHEN OPERATING THE LAUNCHER! NEVER FIRE THE LAUNCHER WHEN SOMEONE IS DIRECTLY IN FRONT OF THE LAUNCHER, NO MATTER HOW FAR AWAY THEY SEEM TO BE! NEVER EVER POINT THE LAUNCHER AT YOUR FACE! VIOLATING ANY OF THESE SAFETY RULES WILL RESULT IN LOST POINTS AND A POSSIBLE EXPULSION FROM THE LABORATORY. HORSEPLAY IS NOT TOLERATED AT ALL! REMEMBER, IT IS ONLY FUNNY UNTIL SOMEONE LOSES AN EYE!

Uniform Circular Motion

The rotating platform used in this experiment is shown in Figure 2. There is a spring on the central rotation axis of the system which provides the force required to keep the brass bob in uniform circular motion. The platform is connected to the Science Workshop computer interface through rotational motion sensor. You will be able to record the angular velocity of the rotation using the computer interface. Your TA will explain how to operate the platform and collect data with computer. Besides the platform you will need a set of different weights.

- Devise an experimental procedure to verify that the force that keeps the bob in uniform circular motion is proportional to the radius of the rotation.

- Devise an experimental procedure to verify that the accelerating force is proportional to the square of the angular velocity of rotation.