- **Emittance-exchange in solenoid channels**
- **Cooling demonstration with asymmetric beam**

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Highlight of our 6D theory

$$\epsilon_s' = -(\eta - ec_-)\epsilon_s + ec_+\epsilon_a + es_+\epsilon_{xy} + b\,\epsilon_L + \chi_s$$

$$\epsilon_a' = -(\eta - ec_-)\epsilon_a + ec_+\epsilon_s + \chi_a,$$

$$\epsilon_{xy}' = -(\eta - ec_-)\epsilon_{xy} + es_+\epsilon_s + \chi_{xy},$$

$$\epsilon_L' = -(\eta - ec_-)\epsilon_L + b\,\epsilon_s + \chi_L,$$

$$\epsilon_z' = -(\partial_\delta \eta + 2ec_-)\epsilon_z + \chi_z,$$

$$e = |\vec{D}| \cdot |\vec{\partial}\eta|/2 \qquad c_\pm = \cos(\theta_D \pm \theta_W)$$

$$e' = |\vec{D}'| \cdot |\vec{\partial}\eta|/2 \qquad s_\pm = \sin(\theta_D \pm \theta_W)$$

$$b = \eta\kappa\beta_T + \alpha_T e s_- + \beta_T e' s_-' \qquad s_-' = \sin(\theta_{D'} - \theta_W)$$

Guidance for developing cooling channel with emittance-exchange

Highlight of our 6D theory

$$
\begin{aligned}
\chi_s &= \frac{1}{2}\beta_T \chi + \frac{1}{2}\mathcal{H}_s\, \chi_\delta, \\
\chi_a &= \frac{1}{2}\mathcal{H}_a\, \chi_\delta, \\
\chi_z &= \frac{1}{2}\beta_L \chi_\delta + \frac{1}{2}\gamma_L (D_x^2 + D_y^2)\chi, \\
\chi_{xy} &= \frac{1}{2}\mathcal{H}_{xy}\, \chi_\delta, \\
\chi_L &= \frac{1}{2}\mathcal{H}_L \chi_\delta.
\end{aligned}
$$

Design principles

- Update good transverse cooling channels
- Maintain periodic structure, esp. beta function
- Create localized dispersion in desired periods

  — closed dispersion bump
- Maximum dispersion at minimum beta
- Keep symmetric focusing
- No dispersion in RF
- Dispersion section to be 1st order achromat

Systems under consideration

Solenoid + dipole + quadrupole + RF + absorber

Lab frame

$$H = \frac{1}{2}\left(p_x^2 + p_y^2\right) + \underbrace{\frac{1}{2}\kappa(s)^2\left(x^2+y^2\right) + \kappa(s)\left(xp_y - yp_x\right)}_{\text{solenoid}}$$
$$\underbrace{-\frac{x\delta}{\rho(s)} + \frac{x^2}{2\rho(s)^2}}_{\text{dipole}} \underbrace{- \frac{1}{2}k(s)\left(x^2-y^2\right)}_{\text{quadrupole}} + \underbrace{\frac{1}{2}\left[\delta^2 + v(s)z^2\right]}_{\text{r.f.}}$$

rotating frame with **symmetric focusing**

$$\tilde{H} = \frac{1}{2}\left(\tilde{p}_x^2 + \tilde{p}_y^2\right) + \frac{1}{2}K(s)\left(\tilde{x}^2+\tilde{y}^2\right) - \frac{\tilde{x}\delta\cos[\theta(s)]}{\rho(s)} - \frac{\tilde{y}\delta\sin[\theta(s)]}{\rho(s)}$$

$$K(s) = \kappa(s)^2 + \frac{1}{2\rho(s)^2}, \quad \theta(s) = -\int_0^s \kappa(\bar{s})\,d\bar{s}$$

Equations for dispersion functions

Dispersion function decouples the transverse and longitudinal motions

$$x = x_\beta + D_x(s)\delta\,,\;\; y = y_\beta + D_y(s)\delta$$

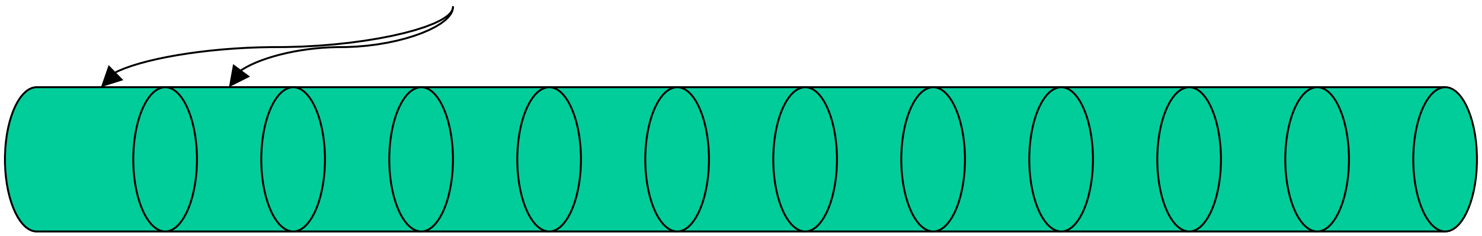
$$\tilde{D}_x'' + K(s)\tilde{D}_x = \frac{\cos[\theta(s)]}{\rho(s)}$$

$$\tilde{D}_y'' + K(s)\tilde{D}_y = \frac{\sin[\theta(s)]}{\rho(s)}$$

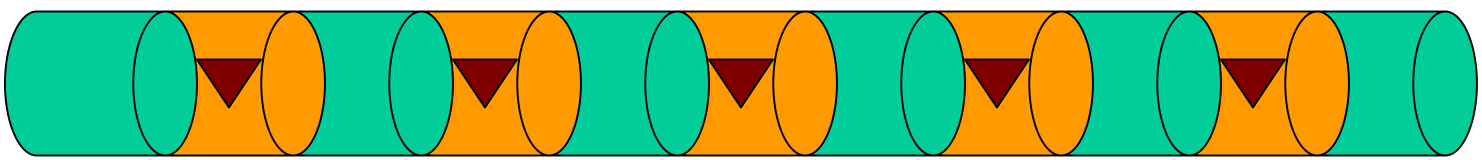
In Larmor frame

Channel layout and specification

- Start with FS-II first cooling cell (5.5m)

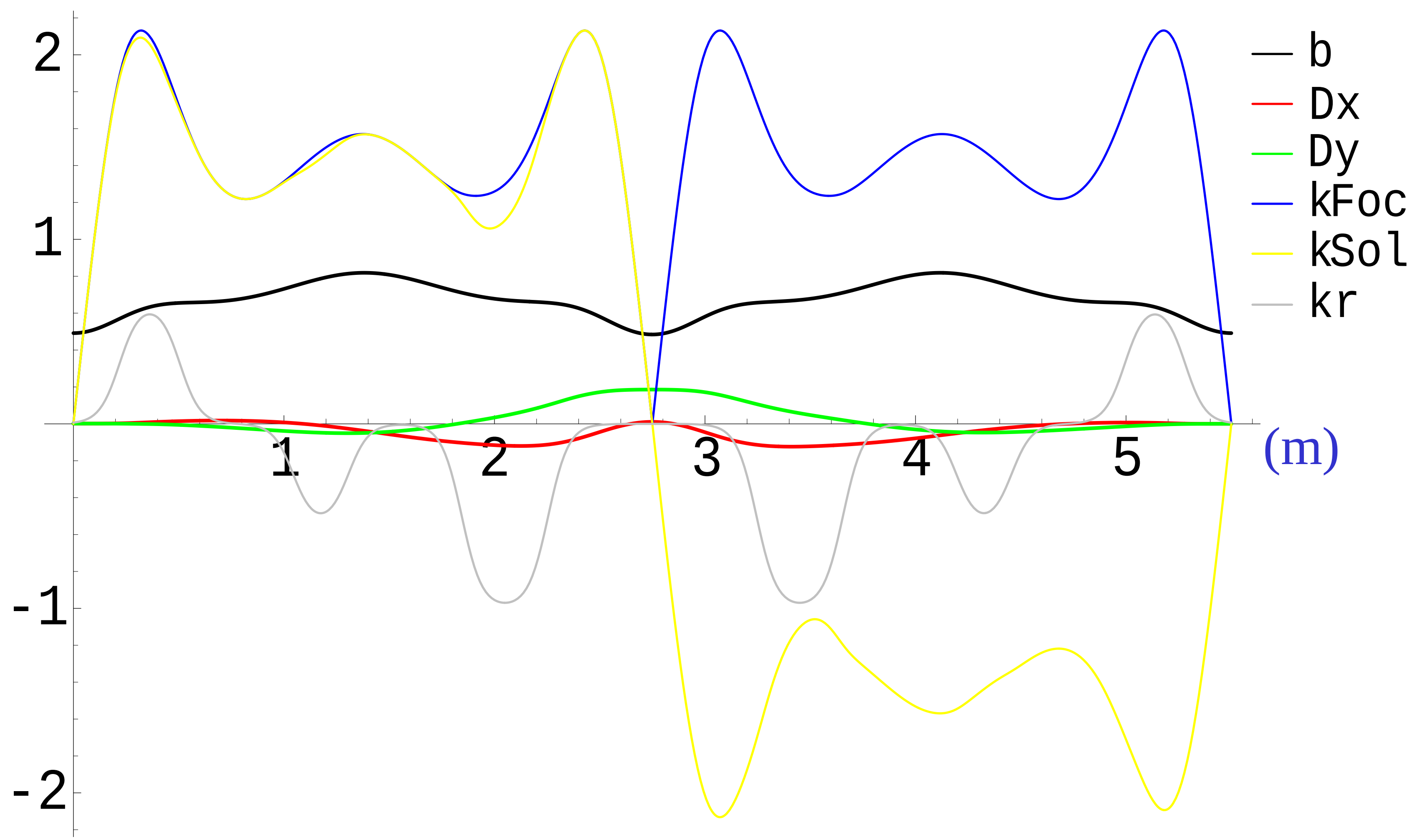


- Replace every other cell with dispersion insertion cell (5.5m)



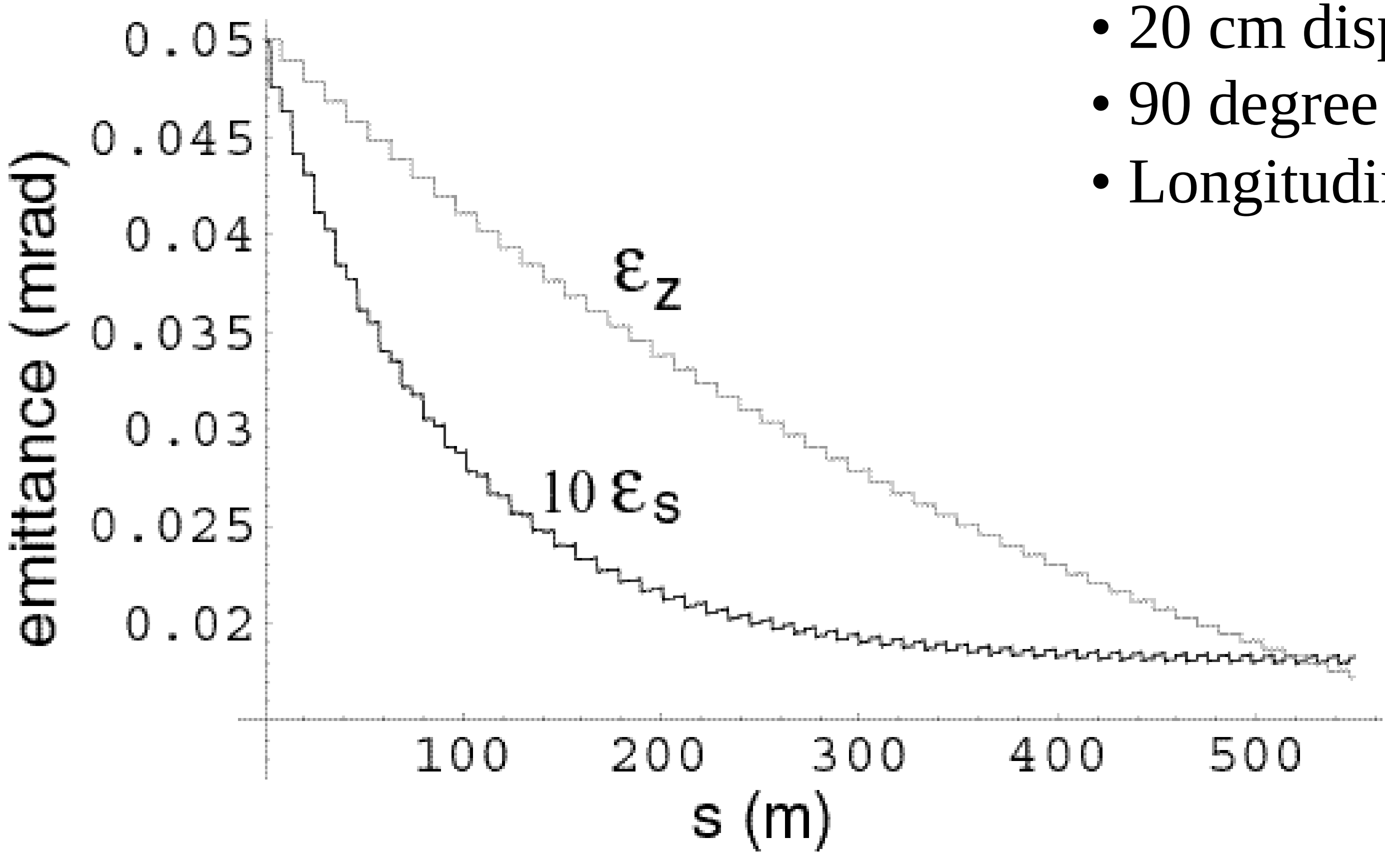
- Dispersion insertion cell is bent-solenoid with symmetric focusing. The actual layout is curved. 90 degree LiH wedges at dispersion maximum. With 20cm dispersion, gives 20% exchange of damping rate. No rf in the dispersion insertion cell.

Example of dispersion insertion



Longitudinal and transverse cooling

based on linear theory



- 20 cm dispersion
- 90 degree LiH wedge
- Longitudinal $\beta \sim \gamma \sim 1$

The longitudinal cooling is very slow. At this point, our focus is to understand cooling dynamics, no optimization at all.

Obstacles to overcome

- **Longitudinal focusing**

  Separation of dispersion and rf means that there is no longitudinal focusing in the dispersion section.

- **Nonlinear dispersion**

  Large aperture and strong magnetic field means that there could be significant nonlinear dispersion, which makes it difficult to control the dispersion at the rf cavities.

- **Slow synchrotron oscillation**

  Limit the amount of damping can be applied, and results in long damping time.

Cooling demonstration goals

- To be demonstrated:

1. cooling principle (untested physics) for CDR funding
2. feasibility (challenging engineering) with CDR funding

- Both are very important. Nice if can be done in one shot, but seems too expensive.
- Could be separated/staged and the different preferences can be better served (if cooling principle can be tested much cheaper) :

1. cooling---prefer relatively long channel
2. engineering---component tests and one integrated cell probably sufficient (at least at beginning). No intense muon beam anyway.

Four generalized beam emittances

$$\epsilon_x \equiv \sqrt{\langle \tilde{x}^2 \rangle \langle \tilde{P}_x^2 \rangle - \langle \tilde{x}\tilde{P}_x \rangle^2} = \frac{1}{2}\langle I_x \rangle$$

$$\epsilon_y \equiv \sqrt{\langle \tilde{y}^2 \rangle \langle \tilde{P}_y^2 \rangle - \langle \tilde{y}\tilde{P}_y \rangle^2} = \frac{1}{2}\langle I_y \rangle$$

$$\epsilon_{xy} \equiv \sqrt{\langle \tilde{x}\tilde{y} \rangle \langle \tilde{P}_x \tilde{P}_y \rangle - \langle \frac{\tilde{x}\tilde{P}_y + \tilde{y}\tilde{P}_x}{2} \rangle^2} = \frac{1}{2}\langle I_{xy} \rangle$$

$$L \equiv \langle \tilde{x}\tilde{P}_y \rangle - \langle \tilde{y}\tilde{P}_x \rangle = \langle L_z \rangle$$

Courant-Snyder type single-particle invariants

$$I_x = \hat{\gamma}(s)\tilde{x}^2 + 2\hat{\alpha}(s)\tilde{x}\tilde{P}_x + \hat{\beta}(s)\tilde{P}_x^2 \,, \text{ etc}$$

Separate cooling from heating

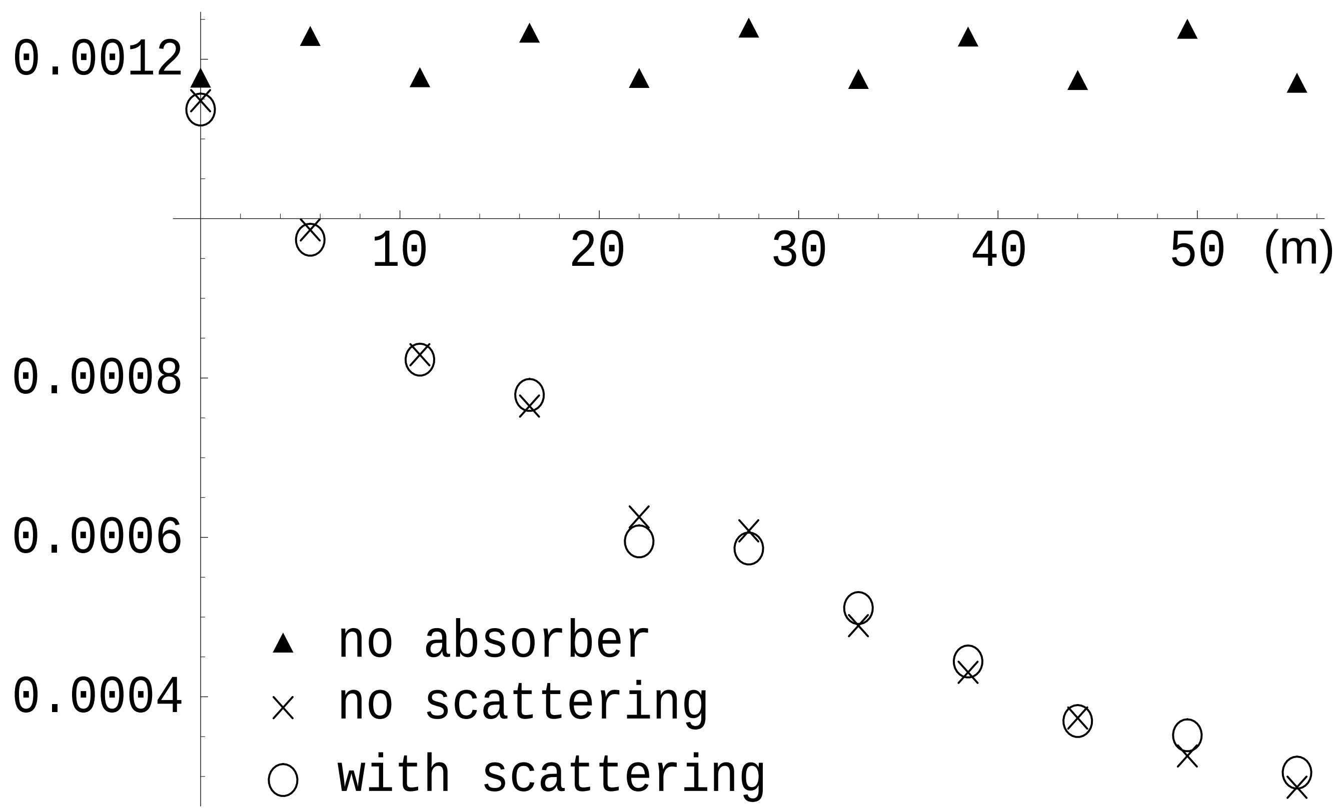
$$\frac{d}{ds}\begin{pmatrix}\epsilon_s\\ L\end{pmatrix} = -\begin{pmatrix}\eta & -\eta\kappa\beta\\ -\eta\kappa\beta & \eta\end{pmatrix}\begin{pmatrix}\epsilon_s\\ L\end{pmatrix} + \begin{pmatrix}\beta\chi\\ 0\end{pmatrix}$$

$$\frac{d\epsilon_a}{ds} = -\eta\,\epsilon_a\ , \qquad \frac{d\epsilon_{xy}}{ds} = -\eta\,\epsilon_{xy}$$

$$\kappa(s) = \frac{qB(s)}{2p_s}, \quad \eta = \frac{1}{p_s v}\frac{dE}{ds}\bigg|_{loss} = \frac{1}{p_s}\frac{dp}{ds}\bigg|_{loss}, \quad \chi(s) = \left(\frac{13.6\,\mathrm{MeV}}{\mathrm{pv}}\right)^2 \frac{1}{L_{Rad}}$$

Note that there is no heating term for the asymmetric emittance, therefore, the cooling effect could be easier to observe in the evolution of an asymmetric beam.

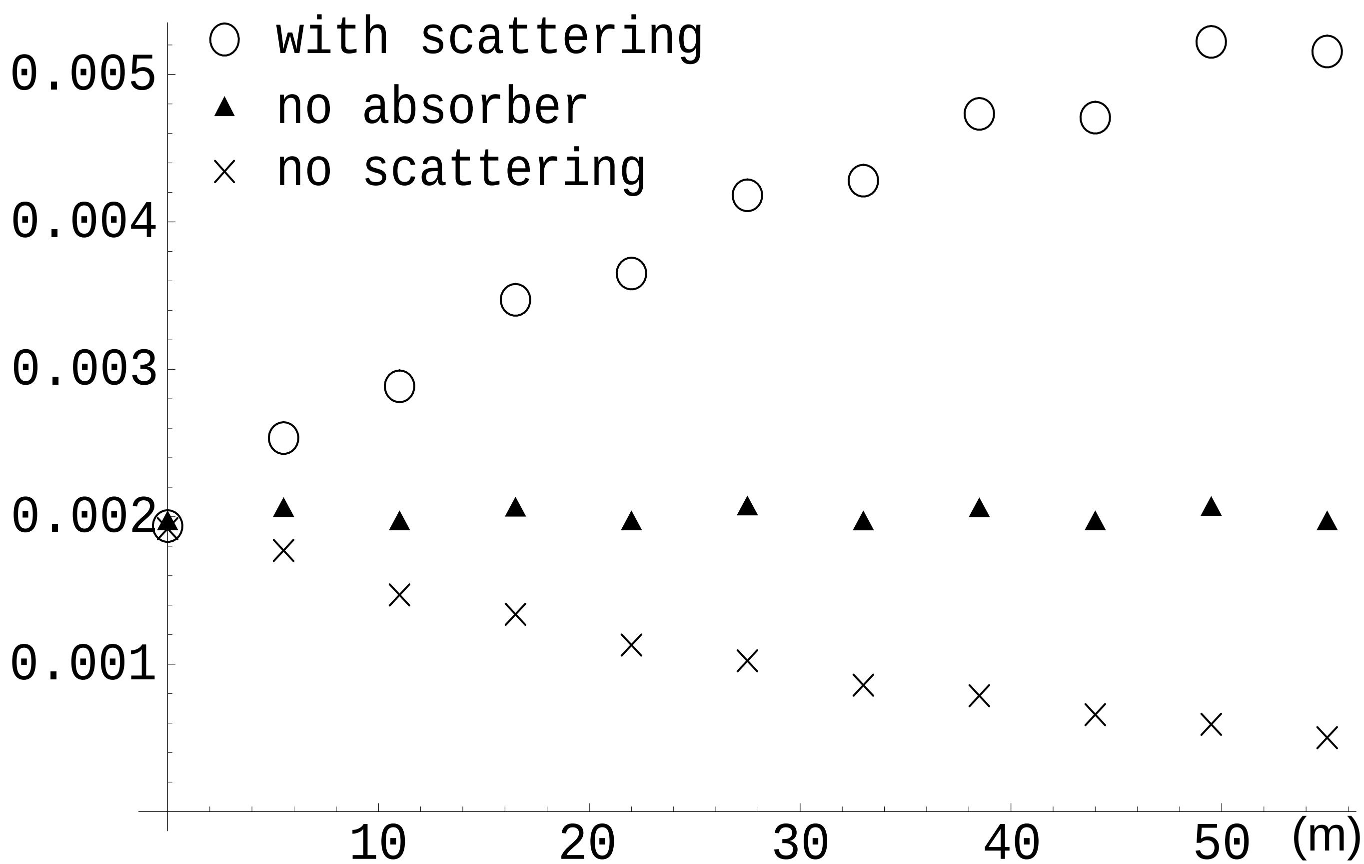
Cooling of asymmetric emittance



ICOOL simulation based on FS-II first cooling cell with input beam parameters:

$\{\sigma x, \sigma y, \sigma z, \sigma px, \sigma py, \sigma pz\} = \{0.01, 0.02, 0.10, 0.0041, 0.0082, 0.005\}$

Cooling of symmetric emittance



ICOOL simulation based on FS-II first cooling cell with input beam parameters:

$\{\sigma x, \sigma y, \sigma z, \sigma px, \sigma py, \sigma pz\} = \{0.01, 0.02, 0.10, 0.0041, 0.0082, 0.005\}$

Measurement of emittances

$$\epsilon_s = \left(\langle x^2\rangle + \langle y^2\rangle\right)/2\hat{\beta}(s),$$

$$\epsilon_a^2 + \epsilon_{xy}^2 = \left[\left(\langle x^2\rangle - \langle y^2\rangle\right)^2 + 4\langle xy\rangle^2\right] \Big/ 4\hat{\beta}^2(s)$$

$$\frac{\epsilon_a}{\epsilon_{xy}} = \tan\left[2\theta_L - \cot^{-1}\frac{2\langle xy\rangle}{\langle x^2\rangle - \langle y^2\rangle}\right]$$

- Reply on measurement of beam profile instead of position.
- Windows in a muon cooling channel provide unique opportunity for such measurements!!

Poor-man's cooling demonstration

- Using Be as absorber instead of liquid hydrogen
- Using larger β-function, lower magnetic field
- Using conventional beam profile monitor instead of spectrometers

The cost of such a channel could be much cheaper, however, the equilibrium emittance could be very large and one probably ends up heating the symmetric emittance. BUT, the cooling effect on an asymmetric beam can still be easily observed and the demonstration contains the same beam dynamics as in the envisioned cooling channels. One may even demonstrate the different cooling rate from different absorbers.

What needed:

- generate asymmetric beam that is matched to the cooling channel.
- good accuracy in beam profile measurements.
- may have to control filamentation.

(Maybe sufficient to compare the evolution of two symmetric beams with different initial emittances. But using one beam could eliminate many uncertainties.)