HomeWork III

Chapter 2
2–4:
First convert to similar units:
\[
 v = \frac{1300 \text{ km/hr}}{hr} \times \frac{1000 \text{ m}}{\text{km}} \times \frac{\text{hr}}{3600 \text{ s}} = 361 \text{ m/s}
\]

Since the ground slopes at 4.3°, the pilot is required to determine the horizontal distance associated with a 10 m rise in the ground.

Since \( \tan \theta = \frac{\Delta y}{\Delta x} \), then
\[
\Delta x = \frac{\Delta y}{\tan \theta} = \frac{35 \text{ m}}{\tan 4.3^\circ} = 465.5 \text{ m}
\]

Since the pilot is travelling at 361 m/s, he/she has:
\[
\Delta t = \frac{\Delta x}{v} = \frac{465.5 \text{ m}}{361 \text{ m/s}} = 1.3 \text{ s}
\]
to react before crashing into the ground.

2-12:
Given:
\[
x = 9.75 + 1.50 t^3
\]
where \( x \) is in cm and \( t \) in s.

a) At:
\[
\begin{align*}
t_i &= 2.0 \text{ s} \Rightarrow x(t_i) = 9.75 + 1.50(2.0)^3 = 21.75 \text{ cm} \\
t_f &= 3.0 \text{ s} \Rightarrow x(t_f) = 9.75 + 1.50(3.0)^3 = 50.25 \text{ cm}
\end{align*}
\]
Then the average velocity is:
\[
\Delta v = \frac{\Delta x}{\Delta t} = \frac{x(t_f) - x(t_i)}{t_f - t_i} = \frac{50.25 \text{ cm} - 21.75 \text{ cm}}{3 \text{ s} - 2 \text{ s}} = 28.5 \text{ cm/s}
\]
b), c), d) The instantaneous velocity is:

\[ v = \frac{dx}{dt} = 0 + 1.50 \cdot 2t^2 = 4.50m/s^3 \cdot t^2 \]

such that

\[ v(2.0s) = 4.5m/s^3(2.0s)^2 = 18.00m/s \]
\[ v(3.0s) = 4.5m/s^3(3.0s)^2 = 40.50m/s \]
\[ v(2.5s) = 4.5m/s^3(2.5s)^2 = 28.13m/s \]

e) The midpoint between \( t_i \) and \( t_f \) is \( x_m = 36.0m \). The instantaneous velocity at which the particle is at \( x_m \) is found by solving

\[ v(t_m) = 4.5m/s^3 \cdot t_m^2 \]

where \( t_m \) is determined from the original equation:

\[ t_m = \left( \frac{x_m - 9.75cm}{1.5cm/s^3} \right)^{1/3} = \left( \frac{36.0cm - 9.75cm}{1.5cm/s^3} \right)^{1/3} = 2.60s \]

such that

\[ v(t_m) = 4.5m/s^3 \cdot t_m^2 = 4.5m/s^3(2.6s)^2 = 30.42cm/s \]

f)
2-15:
a) The quantity \((dx/dt)^2\) is \(v^2\)

b) The quantity \(d^2x/dt^2\) is \(a^2\)

c) The SI units for \(v^2\) is \(m^2/s^2\). The SI units for \(a^2 \) \(m/s^2\).

2-16:
Since acceleration is the slope of \(v\), or
\[
\ddot{a}(t) = \frac{d\dot{v}}{dt}
\]

the acceleration plot looks something like the following diagram. Since the equation is not given, one can only estimate the amplitude, etc.
2-34:
Take the moment of each train applying the brakes to be $t = 0\, s$. Given the constant acceleration, the time that it would take for each train to stop can be determined from $v_f = v_i + at$. Since the problem is in one dimension, I will forgo the vector notation.

First convert velocities to seconds:

$$v_{r_i} = 72\, km/hr \times \frac{1000\, m}{1\, km} \times 1\, hr \times 3600\, s = 20\, m/s$$

$$v_{g_i} = -144\, km/hr \times \frac{1000\, m}{1\, km} \times 1\, hr \times 3600\, s = -40\, m/s$$

Then

$$t_r = \frac{v_{r_f} - v_{r_i}}{a_r} = \frac{(0 - 20)\, m/s}{-1\, m/s^2} = 20\, s$$

$$t_g = \frac{v_{g_f} - v_{g_i}}{g_r} = \frac{(0 + 40)\, m/s}{+1\, m/s^2} = 40\, s$$

such that the green train will still be travelling for $20\, s$ after the red train stops.

The red train will stop at:

$$x_{r_f}(t = 20\, s) = x_{r_i} + v_{r_i}t + \frac{1}{2}a_r t^2 = 0m + (20\, m/s)(20\, s) + \frac{1}{2}(-1\, m/s^2)(20\, s)^2 = 200\, m$$

Likewise, if the green train were to travel the full $40\, s$, it would stop at:

$$x_{g_f}(t = 40\, s) = x_{g_i} + v_{g_i}t + \frac{1}{2}a_g t^2 = 950m + (-40\, m/s)(40\, s) + \frac{1}{2}(1\, m/s^2)(40\, s)^2 = 150\, m$$

Hence, the trains will surely collide!

The speed of the red train will clearly be $v_{r_f} = 0\, m/s$ when the trains collide, since $x_g(t = 20\, s) = 350\, m$. So, only the green train will be moving when they collide. The green train will be at $x = 200\, m$ when it has the velocity:

$$v_{g_f}^2 = v_{g_i}^2 + 2a_g \Delta x$$

such that

$$v_{g_f} = \sqrt{v_{g_i}^2 + 2a_g (x_{g_f} - x_{g_i})} = \sqrt{(-40\, m/s)^2 + 2(1\, m/s^2)(200\, m - 950\, m)} = \sqrt{100\, m^2/s^2} = 10\, m/s$$