HomeWork XI

Chapter 9
9-3:
Given:

\[
m_1 = 3\text{kg} \quad x_1 = 0\text{m} \quad y_1 = 0\text{m} \\
m_2 = 4\text{kg} \quad x_2 = 2\text{m} \quad y_2 = 1\text{m} \\
m_3 = 8\text{kg} \quad x_3 = 1\text{m} \quad y_3 = 2\text{m}
\]

a) The center of mass is given by:

\[
x_{cm} = \frac{1}{M} \sum_i m_i x_i = \frac{m_1 \cdot x_1 + m_2 \cdot x_2 + m_3 \cdot x_3}{m_1 + m_2 + m_3} = \frac{(3\text{kg})(0\text{m}) + (4\text{kg})(2\text{m}) + (8\text{kg})(1\text{m})}{3\text{kg} + 4\text{kg} + 8\text{kg}} = 1.07\text{m}
\]

b)

\[
y_{cm} = \frac{1}{M} \sum_i m_i y_i = \frac{m_1 \cdot y_1 + m_2 \cdot y_2 + m_3 \cdot y_3}{m_1 + m_2 + m_3} = \frac{(3\text{kg})(0\text{m}) + (4\text{kg})(1\text{m}) + (8\text{kg})(2\text{m})}{3\text{kg} + 4\text{kg} + 8\text{kg}} = 1.33\text{m}
\]

c) As the mass, \(m_3\), is increased, the center of mass will approach the position of \(m_3\).

9-15:
No matter when the explosion occurs, the center of mass of the ordinate will follow the trajectory of the initial, unexploded shell. Using a right-handed coordinate system where \(+\hat{i}\) is horizontal to the right and \(+\hat{j}\) is vertical up, the explosion occurs when the \(y\)-component of the velocity is 0. Using:

\[
v_y = v_{y_0} + a_y t = v_{y_0} - gt
\]

then

\[
t = \frac{v_{y_0}}{g} = \frac{|\vec{v}_0| \sin \theta_0}{g} = \frac{(20\text{m/s})(\sin 60^\circ)}{9.8\text{m/s}^2} = 1.77\text{s}
\]

Since there are no horizontal forces and the shell starts from the origin, \((x_0, y_0) = (0, 0)\), the explosion occurs at:

\[
x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2 = |\vec{v}_0| \cos \theta_0 \cdot t = (20\text{m/s})(\cos 60^\circ)(1.77\text{s}) = 17.7\text{m}
\]

\[
y = y_0 + v_{y_0} t + \frac{1}{2} a_y t^2 = |\vec{v}_0| \sin \theta_0 \cdot t - \frac{1}{2} gt^2 = (20\text{m/s})(\sin 60^\circ)(1.77\text{s}) + \frac{1}{2}(-9.8\text{m/s}^2)(1.77\text{s})^2 = 15.3\text{m}
\]

At the very top of the trajectory, there is no vertical velocity. Conservation of momentum tells us that \(p_i = p_f\), where the momentum is purely horizontal. Using \(v_{f_1} = v_{xf}\) and \(v_{f_2} = 0\) to indicate the final velocities of the two fragments after the explosion:
\[ p_i = m v_{x_0} = m |\vec{v}_0| \cos \theta_0 = p_f = \frac{m}{2} v_{f_1} + \frac{m}{2} v_{f_2} = \frac{m}{2} v_{x_f} \]

Then

\[ v_{x_f} = \frac{2m |\vec{v}_0| \cos \theta_0}{m} = 2 |\vec{v}_0| \cos \theta_0 = (2)(20m/s) \cos 60^\circ = 20m/s \]

From the top of the trajectory, the problem can be restated as a projectile of mass \( m/2 \), being launched horizontally from the initial position \( (x_0, y_0) = (17.7m, 15.3m) \). The fragment falls for a time:

\[
\begin{align*}
  y &= y_0 + v_{y_0} t + \frac{1}{2} a_y t^2 \\
  0 &= y_0 + \frac{1}{2} gt^2
\end{align*}
\]

such that

\[ t = \sqrt{\frac{2y_0}{g}} = 1.77s \]

And finally, the \( x \)-position of the final fragment is:

\[ x = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2 = x_0 + v_{x_f} t = 17.7m + (20m/s)(1.77s) = 53.1m \]

9-31:
Using the notation: the mass of the motor is \( M \); the mass of the module is \( m \); the initial speed of the motor-module system relative to earth is \( v_0 \); the relative speed between the motor and the module is \( v_{rel} \); and the speed of the module just after separation relative to earth is \( v \)

Conservation of momentum requires:

\[ (M + m)v_0 = mv + M(v - v_{rel}) \]

Solving for \( v \), and using \( M = 4m \):

\[
\begin{align*}
  (M + m)v_0 &= (m + M)v - Mv_{rel} \\
  (m + M)v &= (M + m)v_0 + Mv_{rel} \\
  v &= \frac{(M + m)v_0 + Mv_{rel}}{M + m} \\
  &= v_0 + \left( \frac{4m}{4m + m} \right) v_{rel} \\
  &= 4300km/hr + \left( \frac{4}{5} \right)(82km/hr) \\
  &= 4365.6km/hr
\end{align*}
\]
9-47:  
(a) Choosing a coordinate system in which the barges are moving in the positive $x$-direction, consider what must happen to the coal that lands on the faster barge during a one minute time interval, $\Delta t = 60s$. In that time, a total of $m = 1000kg$ of coal must experience a change of velocity:

$$\Delta v = v_1 - v_2 = 20km/hr - 10km/hr = 10km/hr \left( \frac{1000m}{1km} \right) \left( \frac{1hr}{3600s} \right) = 2.8m/s$$

(Note: Since the shoveling is taking place perpendicular to the motion of the barges, it does not contribute to the change in momentum.) The force required to change this mass’s velocity on the faster barge in one minute is found from:

$$F = \frac{dp}{dt} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(1000kg)(2.8m/s)}{60s} = 46.7N$$

(b) The problem states that the frictional forces acting on the barges do not depend on mass, so the loss of mass from the slower barge to the faster barge does not affect its motion; i.e. no additional force is required as a result of the shoveling.

9-50:  
(a) The internal energy that the climber must convert to gravitational potential energy is:

$$\Delta U_g = mgh = (90kg)(9.8m/s^2)(8850m) = 7.8 \times 10^6 J$$

(b) The number of candy bars, at $1.25MJ$ per bar, is:

$$n = \frac{7.8MJ}{1.25MJ} = 6.2\text{bars}$$