Chapter 4

Motion in >1 Dimensions
CHAPTER 4

Review: 1-D.

Also, review derivations

Material in this chapter is same as for CH2 except for extending CH2 variables to vectors in >1 dimensions

\[ \vec{F} = \hat{i}x + \hat{j}y + \hat{k}z \]

Then \( \vec{F}_2 - \vec{F}_1 = (\hat{i}x_2 + \hat{j}y_2 + \hat{k}z_2) - (\hat{i}x_1 + \hat{j}y_1 + \hat{k}z_1) \)
\[ = \hat{i}(x_2 - x_1) + \hat{j}(y_2 - y_1) + \hat{k}(z_2 - z_1) \]
\[ = \hat{i} \Delta x + \hat{j} \Delta y + \hat{k} \Delta z \]

Recall that if \( \vec{F}_1 + \vec{F}_2 \) is \( \vec{F} \)

then \( \vec{F}_1 - \vec{F}_2 \) is \( \vec{F} \). Since graphical arithmetic is so simple, check work when working w/ components by using graphical result also. You will have to do this when taking square roots as you'll have 2 solutions!

Do Ex 4.2
Velocity
Just as before:
\[
\langle \mathbf{v} \rangle = \frac{\Delta \mathbf{r}}{\Delta t}
\]
\[
\rightarrow \quad \langle \mathbf{\dot{r}} \rangle = \mathbf{\frac{d}{dt}} \mathbf{r}
\]
\[
= \mathbf{\hat{i}} \frac{d}{dt} x + \mathbf{\hat{j}} \frac{d}{dt} y + \mathbf{\hat{k}} \frac{d}{dt} z
\]
\[
= \mathbf{\hat{i}} v_x + \mathbf{\hat{j}} v_y + \mathbf{\hat{k}} v_z
\]

and

Instantaneous \( \mathbf{v} \)
\[
\mathbf{\dot{r}} = \mathbf{d} \mathbf{r}/d\mathbf{t}
\]
\[
\rightarrow \quad \mathbf{\dot{v}} = \mathbf{\frac{d}{dt}} \mathbf{v}
\]
\[
= \mathbf{\hat{i}} \frac{d}{dt} v_x + \mathbf{\hat{j}} \frac{d}{dt} v_y + \mathbf{\hat{k}} \frac{d}{dt} v_z
\]
\[
= \mathbf{\hat{i}} v_x + \mathbf{\hat{j}} v_y + \mathbf{\hat{k}} v_z
\]

Note that the direction of \( \mathbf{\dot{v}} \) is always tangent to the particle’s path at the instant of the particle’s position.

Ask class Check point 2
Do Example 4.3
Again, use vector arithmetic to manipulate more than one vector.

Acceleration
\[
\langle \mathbf{\dot{a}} \rangle = \frac{\Delta \mathbf{\dot{r}}}{\Delta t} = \frac{\mathbf{\dot{r}}_2 - \mathbf{\dot{r}}_1}{\Delta t} = \mathbf{\hat{i}} \Delta v_x + \mathbf{\hat{j}} \Delta v_y + \mathbf{\hat{k}} \Delta v_z
\]

and
\[
\mathbf{\dot{a}} = \mathbf{d} \mathbf{\dot{r}}/d\mathbf{t} = \mathbf{\hat{i}} \frac{d}{dt} \mathbf{\dot{r}} + \mathbf{\hat{j}} \frac{d}{dt} \mathbf{\dot{r}} + \mathbf{\hat{k}} \frac{d}{dt} \mathbf{\dot{r}}
\]
\[
= \mathbf{\hat{i}} a_x + \mathbf{\hat{j}} a_y + \mathbf{\hat{k}} a_z
\]
\[
\Rightarrow a_x = \frac{d^2 x}{dt^2} \\
\Rightarrow a_y = \frac{d^2 y}{dt^2} \\
\Rightarrow a_z = \frac{d^2 z}{dt^2}
\]
Do Ex 4.4.

Show videos: jersey.neurogen.edu/vlab/cannon

HW - due 13 Feb 4: 1, 6, 9, 10, 14, 17, 21, 26, 47, 52, 59

Do Ex 4.5

Projectile Motion:

0) free fall sec 28

Key concepts:

1) as in all problems - x & y are uncoupled

2) no $\alpha$

3) unless otherwise stated, no drag from air $\Rightarrow$ vacuum

Hence:

$x = x_0 + v_{ox}t + \frac{1}{2}a_xt^2 = x_0 + v_{ox}t = x_0 + 12\frac{v_0^2}{19.1} \cos \theta t$

$y = y_0 + v_{oy}t + \frac{1}{2}ayt^2 = y_0 + v_{oy}t - \frac{1}{2}gt^2$

- x is constant v, it will continue until it runs out of t.

If $y_i = y_f$, then $R = x - x_0 = v_{ox}t = 12\frac{v_0^2}{19.1} \cos \theta t$

use y to find t

$y - y_0 = 0 = v_{oy}t + \frac{1}{2}gt^2$

$\frac{12\frac{v_0^2}{19.1} \sin \theta t - \frac{1}{2}gt^2}{t = \frac{2v_0 \sin \theta}{g}}$

Substitute:

$R = \frac{2\frac{v_0^2}{19.1} \sin \theta \cos \theta}{g} = \frac{12\frac{v_0^2}{19.1} \sin \theta}{g}$

Then $R_{max}$ is when $\sin 2\theta = 1 \Rightarrow 2\theta = 90^\circ \Rightarrow \theta = 45^\circ$.
Have to do derivation

\[ x = x_0 + v_{0x}t + \frac{1}{2} a t^2 \]

\[ y = y_0 + v_{0y}t - \frac{1}{2} g t^2 \]

\[ v_y = \frac{dy}{dt} = v_{0y} - gt \]

\[ v^2 = (v_y - \frac{1}{2} v_{0y} \cos \theta)^2 - \frac{1}{4} v_{0y}^2 \sin^2 \theta \]

Then,

\[ \sqrt{v^2} = \frac{1}{2} v_{0y} \cos \theta + v_y - 2g \frac{v_{0y} \sin \theta}{g} \]

Then,

\[ v_y = \frac{1}{2} v_{0y} \cos \theta + v_y - 2g \frac{v_{0y} \sin \theta}{g} \]

\[ y - y_0 = \frac{v_{0y}^2}{g} - \frac{1}{2} g t^2 \]

\[ = \frac{v_{0y}^2}{g} \left( \frac{1}{2} \sin^2 \theta - v_y^2 \right) \]

\[ = \left( \frac{1}{2} \sin^2 \theta - v_y^2 \right) - \frac{1}{2} \frac{1}{g} \left( v_{0y} \cos \theta - v_y^2 - 2g \frac{v_{0y} \sin \theta}{g} \right) \]

\[ = \frac{1}{2g} \left[ \frac{1}{2} v_{0y}^2 \sin^2 \theta - \frac{1}{2} \frac{1}{g} \left( v_{0y} \cos \theta - v_y^2 - 2g \frac{v_{0y} \sin \theta}{g} \right) \right] - \frac{v_y^2}{2g} \]

So,

\[ 2g (y - y_0) = \frac{v_{0y}^2}{g} - v_y^2 \]

or

\[ v_y^2 = \frac{1}{2} v_{0y}^2 \sin^2 \theta - 2g (y - y_0) \]


Uniform Circular Motion

Acceleration is change of velocity vector - direction and/or speed

\[ a \]

Fig. 4.18: shows differences in \( v \) direction
\( \vec{v} \) is tangent to change in position
\( a_{\|} \Rightarrow \text{change of speed} \) recall \( \vec{a} \) is vector
\( a_{\perp} \Rightarrow \text{change of direction} \)
So, w/UCM \( \vec{v} \) is constantly changing (not in \( \vec{r} \))
\[ a_{\|} = 0 \]
\[ a_{\perp} \neq 0 \]
\( a_{\perp} \) points in or out of circle - in order to stay in circle - must be "in"
\[ \Rightarrow \text{toward center} \]

\( a = \frac{v^2}{r} \) where \( v = |\vec{v}| \)
\( r \) is radius of circle

- Path travelled is \( 2\pi r \)
\[ \Rightarrow v = \frac{2\pi r}{T} \]
or \( T = \frac{2\pi r}{v} \) period

Motion is "Periodic" - will see this term again

- Show proof on p. 61
- Ask class check point 6
- Do Example 4.9
Relative Motion

\[ (17) \]

(One frame as seen from another)
Only for constant \( \dot{v} \)
relative motion, i.e. \( \dot{a} = 0 \)

1) Frames of reference
- scooter example
- note that if \( B \) moves wrt \( A \), then from \( B \)'s reference frame \( A \) is moving backwards at \( \dot{v}_{BA} = -\dot{v}_{AB} \)
- scooter w/32D object

2) \[ \begin{align*}
\text{Relative Position} \\
X_{AP} = X_{BP} + X_{AB}
\end{align*} \]

at an instant in time, say \( t \)

\[ \begin{align*}
\text{Relative Velocity} \\
\dot{X}_{AP} = \dot{X}_{BP} + \dot{X}_{AB}
\end{align*} \]

\[ \begin{align*}
\dot{X}_{AP} &= \dot{v}_{AP} \\
\dot{v}_{AP} &= \dot{v}_{BP} + \dot{v}_{AB}
\end{align*} \]

Note that

\[ \dot{v}_{BP} = \dot{v}_{AP} - \dot{v}_{AB} \]

\[ \begin{align*}
\text{Acceleration of } P \\
\ddot{X}_{AP} = \ddot{X}_{BP}
\end{align*} \]

\[ \begin{align*}
\ddot{v}_{AP} &= \ddot{v}_{BP} + \ddot{v}_{AB} \\
\alpha_{AP} = \alpha_{BP}
\end{align*} \]

\[ \text{sind } \frac{\Delta t}{\Delta t} = 0 \]

Figure 6: