Physics 221
Experiment 1: Harmonic Motion

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Introduction

Newton’s Laws are evidently at work in simple linear motion, but the behavior of bodies is almost never that simple. Newton’s
Laws are also applicable to the study of harmonic motion. Harmonic motion is any type of periodic motion that repeats itself
exactly after a certain time interval—the period. The most fundamental type of this motion is known as Simple Harmonic Mo-
tion (SHM). SHM occurs frequently throughout physics—not only in mechanics, but also in electromagnetism and quantum
physics. In fact, many types of oscillations and vibrations found in nature can be modeled as SHM, or one of its extensions.
In this experiment, we will study simple harmonic motion using pendulums and springs.

Oscillating Mass on a Spring

When a system undergoes a simple harmonic motion, it exhibits a continuous, smooth, and periodic motion which remains
unchanged over a longer period of time. For example, recall that the force produced by a spring is:

\[ F = -kx \]  

where \( k \) is the spring constant and \( x \) is the length of extension (positive \( x \)) or compression (negative \( x \)). This equation (known
as Hooke’s Law) can be rewritten as

\[ ma = -kx \]

where we have used Newton’s second law. Using the usual dot convention to denote time derivatives, we can then write

\[ a = \frac{dv}{dt} = \ddot{v} = \frac{d^2x}{dt^2} = \ddot{x} \]

Equation 2 then becomes

\[ \ddot{x} = -\frac{k}{m}x = -\omega^2 x \]  

where we have defined the constant \( \omega = \sqrt{k/m} \). One may show that the solution \( x(t) \) is sinusoidal, with one form of the
solution being

\[ x(t) = A \cos(\omega t + \delta) \]  

where \( A \) is the amplitude of the oscillation and \( \delta \) is the phase of the oscillation. If we pull the mass \( m \) to stretch the spring by
a distance \( A \), and then release it from rest, the initial conditions on the motion (at \( t = 0 \)) are

\[ x(0) = A; \quad v(0) = 0; \quad \delta = 0 \]

Applying these initial conditions to Equation 4 gives the equation of motion of the spring-mass system as

\[ x(t) = A \cos(\omega t) \]

Note that if we had instead written the solution of Equation 3 as \( x(t) = A \sin(\omega t + \delta) \), then for the same initial conditions, the
phase would need to be set to \( \delta = \pi/2 \).
The parameter \( \omega \) that we used is the angular frequency of oscillation. Therefore, the frequency of oscillation of the mass is

\[
f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]  

(7)

and the period of oscillation is

\[
T = \frac{1}{f} = 2\pi \sqrt{\frac{m}{k}}.
\]  

(8)

Using the solution given in Equation 6, the velocity of the oscillating mass is

\[
v(t) = \dot{x}(t) = -\omega A \sin(\omega t)
\]  

(9)

Note that the maximum values of \( x(t) \) and \( v(t) \) occur at phases that differ by \( \pi/2 \). That is, when \( x(t) \) has its maximum value, \( v(t) \) is zero and vice versa. The energy of a mass on a spring as a function of time can then be written as

\[
E = \frac{1}{2} k x^2 + \frac{1}{2} m v^2 = \frac{1}{2} k A^2 \cos^2(\omega t) + \frac{1}{2} m A^2 \sin^2(\omega t)
\]  

(10)

\[
= \frac{1}{2} k A^2 \left[ \cos^2(\omega t) + \sin^2(\omega t) \right]
\]  

where we have used the trigonometric identity \( \sin^2 \theta + \cos^2 \theta = 1 \). Since both \( k \) and \( A \) are constants, this shows that the energy in a simple harmonic oscillator is also a constant, regardless of the position and velocity of the mass. The mass-spring system is closed, and no energy is lost. The motion then meets the conditions of SHM, with continuous periodic motion.

### Oscillating Mass on a Pendulum

According to popular myth, Galileo felt bored one day in church. He looked up and saw that the light fixtures attached to the ceiling of the church swayed back and forth in regular motion. Thus began the first quantitative analysis of the motion of a simple pendulum. An "ideal" or "simple" pendulum is constructed with one end of a "massless" string attached to a small, massive bob, the other end being pivoted about a fixed location. The force on the pendulum bob can be expressed in terms of \( \theta \), the angle of the string with respect to the vertical (equilibrium) position:

\[
F = mL \ddot{\theta} = -mg \sin \theta.
\]  

(11)

where \( L \) is the length of the string. (see Figure 1).

Equation 11 is not as trivial to solve as for a mass on a spring. However, we may expand \( \sin \theta \) in powers of the angle \( \theta \) (where \( \theta \) is measured in radians):

\[
\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - ...
\]

and then make the approximation for a small angle oscillation (\( \theta \ll 1 \)), keeping only the first (linear) term, \( \sin \theta \approx \theta \). Substituting this into Equation 11, we obtain

\[
L \ddot{\theta} = -g \theta
\]  

(12)

\[
\ddot{\theta} = -\frac{g}{L} \theta
\]
This equation is then identical in form to Equation 3, and we may solve it in the same way. In particular, we can make the identification that for the simple pendulum, \( \omega^2 = \frac{g}{L} \), so that the period of oscillation for a simple pendulum is

\[
T = 2\pi \sqrt{\frac{L}{g}}
\]

(13)

**Part A : Vertical Spring Motion**

**Procedure :**

1. Hang one end of the spring on a ring stand.
2. Attach a mass \( m \) to the lower end of the spring.
3. Displace the mass a small distance and let it oscillate.
4. Record the time \( t \) for the mass to make 10 complete oscillations.
5. Repeat steps 3 and 4, for a total of four recorded times.
6. Repeat the experiment with different masses, corresponding to five different sets of measurements. Your raw data table should consist of a column for the five masses, and columns for the times.

**Analysis :**

1. Add another column to your raw data table consisting of the average period of oscillation, \( T \), computed from your measurements, and another column for \( T^2 \). Keep in mind that the recorded time \( t \) equals \( 10T \).
2. Plot \( T^2 \) versus \( m \).
3. Using Equation 8, derive an expression for the slope of the graph of \( T^2 \) versus \( m \). Using the best-fit line from your graph, determine the spring constant \( k \).
4. Using Hooke’s Law (Equation 1), check to see if your spring constant value makes sense. If it does not, go back and check your work.

**Part B : Simple Pendulum**

Refer to Figure 1 above. For this part, a simple pendulum and a Pasco timer will be used. The parameter to be varied is the length of the string, \( L \).
Procedure:

1. Attach the free end of the string to the ring stand.
2. Measure $L$, the distance from the point of attachment to the center of mass of the bob.
3. Release the bob from a small angle (approximately five degrees from its rest position), and time ten oscillations. Record the total time $t$ and $L$. Repeat several times, for a total of four observations.
4. Repeat this procedure for a total of five different lengths. The data should be placed in one raw data table containing columns for $L$ and the times.

Analysis:

1. Find the average period of oscillation, $T$, and $T^2$. Add these values to your raw data table. Again, keep in mind that the recorded time $t$ equals $10T$.
2. Plot $T^2$ versus $L$.
3. Using Equation 13, derive an expression for the slope of a $T^2$ versus $L$ graph. Using the best-fit line from your graph, determine the value of $g$ and compare it to the accepted value. Explain why there is a difference. (If your value differs significantly, go back and check your calculations.)

Part C: Introduction to the Oscilloscope

Using an oscilloscope is an extremely useful skill, so it is important that you have the time to study the instrument alone, without having to worry about other additional physical concepts in an experiment as well. Although this portion of the experiment looks lengthy, the procedure is very basic, straightforward, and short.

The following is a detailed description of most of the functions and features of an oscilloscope. It would be ideal to read the section carefully before coming to laboratory, and then go through the procedure steps while using the oscilloscope to look at very simple voltage signals. In your oscilloscope write-up, describe what you performed in the laboratory in a clear and concise manner, while answering the questions given in the procedural steps.

The oscilloscope is the most useful and versatile electronic instrument. It acts as a voltmeter with many added capabilities. Instead of just measuring a numerical voltage value at a point in a circuit, it allows you to follow the changes of voltage as a function of time. Thanks to a “trigger” mechanism, voltage signals can be displayed as stationary waveforms on a screen.

For much of the 20th century, oscilloscopes were constructed with analog circuitry and cathode-ray tube (CRT) displays, but the oscilloscopes you are likely to encounter will be digital, with LCD displays rather than CRTs. However, the controls on a digital oscilloscope are based on those of an analog oscilloscope, so it important to understand how an analog oscilloscope operates. A very basic description of the insides of an oscilloscope follows. An electron gun produces a beam of electrons, which is focused on a fluorescent screen. The beam travels through two sets of deflection plates; one set deflects the beam in the $x$-direction, the other set deflects the beam in the $y$-direction. The potential difference between the deflection plates is determined by the voltage signal entering the oscilloscope input. This potential difference causes the beam to be deflected up or down by a certain amount proportional to the input voltage. Thus, the oscilloscope acts a voltmeter on the $y$-axis. If another voltage is generated to ramp repeatedly, and applied to the $x$ deflection plates, the light beam will be repetitively deflected across the screen. This allows variations in time of the incoming voltage signal to be seen. This ramping voltage applied to the $x$ deflection plates is called the sweep.

A typical front panel of an oscilloscope can be seen in Figure 2. The most obvious feature of the oscilloscope is a screen divided into many squares by an overlying grid (for a digital oscilloscope, the grid only appears when the device is on, of course). The vertical axis on the grid represents the voltage of an incoming signal, while the horizontal axis, in most cases, represents time. Most oscilloscopes have two input channels; incoming voltage signals enter the oscilloscope through these channels. Having two input channels is useful because it allows the user to look at two different signals (e.g. at different points of a circuit) at one time. Under the VERTICAL heading, each channel has a knob labeled VOLTS/DIV. Turning this knob will change the vertical scale of the displayed signal. The knob labeled POSITION moves the voltage signal display up and down on the screen. Each channel also has a MENU button that brings up the channel menu, and turns the waveform display on or off.
The controls under the HORIZONTAL heading perform other functions. As mentioned previously, the horizontal direction on the oscilloscope represents time. The SEC/DIV setting determines the amount of time displayed. For example, if the setting is 0.1 ms, this means that the oscilloscope is sweeping at a rate such that each square or division represents 0.1 millisecond of time. Similar to the vertical controls, there are many numerical settings to accommodate a large range of incoming voltage signal frequencies.

The trickiest part about the oscilloscope is "triggering." So far we have discussed vertical signals and horizontal sweep: those are what we need for a display of voltage versus time. But if the horizontal sweep doesn’t catch the input signal at the same time (assuming the signal is repetitive), the display will be a mess—a picture of the input waveform superimposed over itself at different times. Another result of improper triggering is the voltage signal “moving” on the screen, rather than being stationary so that measurements can be made. The TRIGGER MENU contains a number of different settings about trigger sources and modes. Setting the triggering control on AUTO mode should be satisfactory for our purposes. If this doesn’t work, you can try the NORMAL mode while adjusting the LEVEL knob. More details about these controls will be discussed in a more advanced course.

Procedure:

We will acquaint ourselves with the oscilloscope front panel controls, and then make some observations. We will observe a DC voltage signal from a power supply, and an AC voltage signal from a function generator.

1. With nothing connected to the front of the oscilloscope, find the power switch and turn it on. Wait for the oscilloscope to “boot up” and show the grid, then press the AUTOSET button near the top right. After a few seconds, the display should show two fuzzy horizontal lines. These lines are a "picture" of the voltage over a certain amount of time—each channel shows zero since nothing is connected. Press the CH1 MENU button several times and observe how the waveform turns on and off. Note the “1” at the left of the waveform display; this marks the zero voltage level for channel 1. Use the channel 1 POSITION knob to move the display up and down. Note the right side of the display; these menu options change when different menus are selected and are controlled by the unlabeled buttons closest to the screen. With the CH1 menu set, make sure the following settings are selected: [Coupling: DC, BW Limit: OFF, Volts/Div: Coarse, Probe: 1X]. Select the CH2 menu and make sure the settings are the same.

2. Connect the sine wave output of the function generator (Audio Generator) to channel 1 of the oscilloscope. You will need to use a BNC to banana jack adapter. The female end of a BNC connector is the connection on the front of the oscilloscope. The male end of a BNC connector is the attachment that physically twists on to the female end. If you look at the male end, there is a small pin in the very center. This pin is connected to the wires that will carry the voltage signal from the function generator. The rest of the connector is physically touching the case of the oscilloscope, which is usually tied or connected to ground. BNC connectors need to be lined up in a particular direction to make an attachment; do not attempt to force them. The banana jacks are color coded: red for signal/voltage and black for ground.
3. Now, put a DC voltage signal from the power supply into channel 2. You will need another BNC to banana jack adapter. The oscilloscope should now be connected properly to the function generator and the power supply.

4. Turn on the power supply and function generator. Set the function generator frequency to 1500 Hz. On the oscilloscope, use the SEC/DIV knob to set the horizontal scale to 1 ms/DIV, and adjust the CH1 VOLTS/DIV knob until a sine wave is visible on the display. Note that the lower left of the display shows the volts per division setting for each channel, and the center bottom shows the time per division. If the signal is small and noisy, you should increase the amplitude setting on the function generator. If you do not see a stationary sine wave, you may need to adjust the trigger settings in order to stabilize the display, or press the AUTOSET button.

5. Turn off the CH1 waveform display and make sure CH2 is still on. Set the power supply to 5 V and adjust the CH2 VOLTS/DIV setting until a line appears on the screen. Use the CH2 POSITION knob to move the “2” at the left of the screen to the middle of the display; note that a status line appears at the very bottom of the display telling you how far off you are. Find the VOLTS/DIV setting that positions the signal at the first grid line above the center line. Now slowly increase the voltage output on the power supply to 10 V. What happens to the line on the screen? Change the VOLTS/DIV settings in both directions and describe what happens.

6. The oscilloscope is normally operated in DC-coupled mode, which means you can see AC voltage signals on top of constant, DC voltage signals. Sometimes you may want to see a small AC signal, perhaps electrical "noise", riding on a large DC voltage. In this case, you can change the Coupling setting in the CH1/CH2 menu to AC. What happens to the DC voltage display when the CH2 coupling is changed to AC?

7. Turn off the CH2 display and turn CH1 back on. You are now looking at the AC voltage waveform coming from the function generator. Adjust the oscilloscope so that only a few cycles of the sine wave appear on the screen. This can be done by changing the value of the SEC/DIV setting. How would you determine the frequency of the sine wave using the oscilloscope? Here are a few hints. Remember that you can horizontally move the signal (i.e. in time) with the horizontal POSITION knob. This will enable you to align the peaks of the sine wave with the grid lines so as to determine how long it takes for one cycle to be completed. Also, the SEC/DIV settings can be changed so that it may be easier to align the peaks of the waveform to the grid, or fit several complete cycles of the waveform between the grid lines. Using one or both of these methods, determine the frequency of the sine wave using the oscilloscope.

8. Press the CURSOR button to bring up the CURSOR menu. Set the Type to Time and the Source to CH1. In CURSOR mode, dotted lines appear on the screen that can be used to measure either voltage or time, and the POSITION knobs control the position of the cursors. Position the cursors to mark out one period of the sine wave. Write down the Delta (the distance between the cursors) and calculate the frequency.

9. Adjust your SEC/DIV and VOLTS/DIV settings so that several cycles of the sine wave are visible. Now press the MEASURE button to bring up the MEASURE menu. You may need to change the Source and Type settings in order to select CH1 Freq, which will produce a measurement of the frequency. Compare this frequency value to 1) the value indicated on the function generator, 2) the value found using cursors, and 3) the value found by estimating visually. They should all be roughly the same. Which value would you deem to be most accurate?

10. The amplitude of the signal now needs to be determined. Write down the amplitude and peak-to-peak values for the wave 1) estimating visually, 2) using cursors, and 3) using the MEASURE menu (peak-to-peak only). Discuss the differences between the results, if any.